Decimals, DENOMINATORS, DEMONS, calculators and *connections*

LEN SPARROW and **PAUL SWAN** provide some practical activities for overcoming some fraction misconceptions using calculators specially designed for learners in primary years.

It may be a coincidence and have no relevance at all but have you noticed that the word denominator starts with the same letters as the word demon only in a mixed up form. For many children the world of the denominator is also mixed up and brings forward the demons of misunderstanding, confusion and fear, which remain with them for the rest of their lives.

There is an array of reasons for the demons, as a quick survey of the research literature will show (see for example Booker, 1998; Newstead & Murray, 1998). In the areas of fraction, including decimal fraction, teaching a variety of "quick fix" rules abound — to multiply by ten you add a nought; turn the second fraction upside down and multiply. Generally, these lead to a long-term confusion, misapplication and a limited view of mathematics as merely remembering formulae and rules.

The calculator as a learning aid

One of the ways to move beyond procedural teaching and learning into developing conceptual understanding is to use one of the familiar tools of society — the calculator. When used in sensible ways, as part of a broad teaching package, the calculator can allow children to enter a world of understanding and emerge into adult life without the demons.

In our view, the calculator, when used in sensible

ways (see Sparrow & Swan, 2000), has the potential to be a powerful teaching and learning aid, and something to challenge and excite children in mathematics. For most children, using the calculator in mathematics teaching will generate motivation, interest and possibly reduce the chorus of groans that often accompany the announcement that it is time for mathematics.

The calculator is not an electronic answer book for checking work, nor an easy option for cheating and no thinking. It is, in fact, if used in the ways we will suggest, a machine to engage children in thinking about mathematics.

A justification for our use of calculators with children in mathematics classes mainly relates to their embodiment as a powerful learning and teaching tool, much in the way teachers might use MAB (Base 10 blocks) to develop mathematical understanding. By engaging with a calculator as part of their mathematics learning, children are learning about and using the tools of society as well as developing a deeper understanding of mathematics. They are learning with the aid of technology, becoming techno-literate (Sparrow & Swan, 2005) as well as developing number sense. In fact, it is often one of our aims to have children use a calculator to understand an aspect of mathematics in such a way that in future they will not have to use a calculator to perform the same piece of mathematics.

A more function calculator for older primary children

The idea of the model of the calculator developing in complexity and number of functions (see Figure 1) as children become older has been explored elsewhere (Kissane, 1997; Sparrow & Swan, 2000).

Growth in age →

Younger children	Older primary children	Secondary school students
Four function calculator	More function calculator	Multi-function calculator
TI-108	TI-15	TI-83

Growth in calculator functions →

Figure 1. The growth of calculator function complexity.

Figure 2. Four function calculator (TI-108).

Figure 3. More function calculator (TI-15).

Planning with the calculator available

Another reason to select a calculator is to consider its potential for supporting the particular learning you are planning to introduce. In the case of fractions with older primary children, the simple four-function calculator found in many classroom cupboards is limited in its scope. The more function TI-15 is better suited to supporting the planned tasks. It has functions that will present fractions in a "stacked format", simplify fractions, convert common fractions to decimal fractions and decimal fractions to common fractions, perform fraction calculations for addition, subtraction, multiplication and division, work with mixed fractions and improper fractions, and "round" numbers to a range of decimal places.

	Decimal fractions	Common fractions
Four function	OK but rather limited in use	Very limited use
TI-108	The more functions will allow greater scope for tasks	Has extended fraction functions that can be used to help children's learning

Figure 4. Selecting the model of calculator.

Calculator available activities for learning fraction ideas

A question asked by many people relates to the fact that there is nothing left to teach if the calculator can perform all the calculations required of children in the older primary years. We are using the availability of a powerful calculator here to help children understand and develop a deeper concept of fraction ideas. We are helping children build conceptual understanding as well as understanding the procedures involved with calculations and fractions (both common and decimal notations).

In all the activities and games suggested below it is vitally important that children are required to explore ideas and explain their thinking and methods. Discussion at the end of the activity or even during it is essential to make explicit the mathematical purpose for the task and to help children connect this new knowledge to what they already know. Just giving children calculators has little or no potential for learning mathematics and may lead to the images of non-thinking children offered by opponents of calculator use in schools.

Decimal fractions

One of the problems many children have is with the over-generalisation of rules without fully understanding the particular ideas behind them. The use of the rule add a nought when multiplying by ten is an example. For a number of years children will have added noughts to whole numbers and will have gained correct answers, for example $6 + 0 = 6$ where the answer does not change from the original. Later, they will be given the add a nought rule in a different context of multiplying by ten. Here the rule application is in conflict with previous teaching. Now the answer does change from the original: $6 \times 10 = 60$; $72 \times 10 = 720$. As they move into the area of decimal numbers, the rule begins to break down. For example, when presented with 3.5×10, many children apply the add a nought rule and produce an incorrect answer of 3.50. Others add a nought to the whole number producing another incorrect answer of 30.5, while others add a nought to both numbers and generate 30.50 A quick rule given without understanding in the early years may result in misapplication later.

Multiplying by ten with a calculator

The calculator is used here to generate lots of data quickly. The important part of the task is the "maths noticing" with the help of the teacher or task partner.

The use of a chart (see Figure 5) is important in this instant as it makes visible the key presses and the answers gained from using the calculator. It acts as a focus for the later discussion between children and teacher. On most calculators the numbers and calculations disappear and are not available for discussion as children press further keys. The TI-15 is unlike most calculators in the primary classroom as it has a larger display and a function that allows a "history" of key pushes to be viewed.

Children can select whole numbers less than 100 in the Start number column. The number is multiplied by 10 on the calculator and the Display number is recorded. The sequence is repeated at least five times. Children then start with decimal numbers less than one, for example 0.3 and follow the same sequence. The Display numbers may be in conflict with what they are expecting. This is a useful place for discussion about what is happening and what they are noticing. A "multiplier" (Booker, Bond, Sparrow & Swan, 2004) is a useful teaching aid to help children "see" the rule of moving digits one place to the left in relation to the decimal point (Figure 6). The "nought" in this case acts as a placeholder to show the correct number of place value columns in the answer.

Start number	Multiply by 10	Display number	Comment
45	× 10	450	

Figure 5. Multiplying by ten recording table.

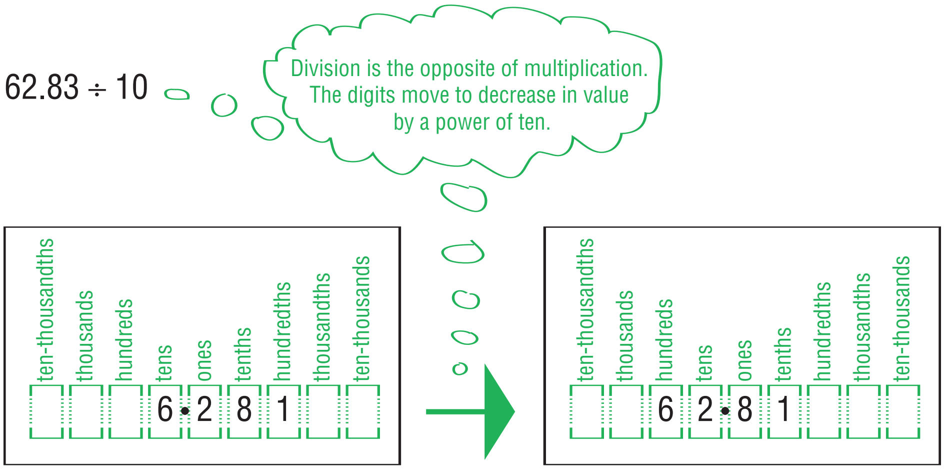

Figure 6. Example of a multiplier.

Calculators can be used to test large numbers or numbers with lots of decimal places to see if the rule always works. The task can be developed to consider rules for multiplying by 100 and 1000 or for dividing by 10, 100 and 1000.

Make it zero again

Many children will have experienced using a calculator and the task *Wipe out* (Sparrow & Swan, 2001) or an activity with a similar name, where digits in a number are reduced to zero. The same format can be used with older children and decimal numbers. The task is also a useful way to practise the rule highlighted in the previous task.

The task starts with the children keying into their calculators a decimal number, such as 123.45. They are asked to complete the table as shown in Figure 7 (or record the display "history" on the calculator). This time the *wipe out* rules are changed to state that the digits may only be wiped out to zero in the "ones column". For the 3, subtracting 3 quite easily achieves this. The *display number* now becomes the *start*

Start number	Operation	Display number	Comment
123.45	– 3	120.45	Wipes out the 3
120.45			

Figure 7. Wipe out recording chart.

number 120.45. Children now have to "move" a digit to the "ones column" for it to be wiped out. If the number is multiplied by 10 the 4 will "move" to the ones column

120.45 × 10 = 1204.5

The task continues by applying the multiply by 10 or divide by 10 rules to "move" the digits to the "ones column". Children could also be challenged to apply the multiply and divide by 100 or 1000 rules to "move" the digits if the teacher does not allow the use of multiply or divide by ten rule.

Fraction notations

Remainders, common fractions and decimal fractions

Often children mistake a remainder after a division operation with the decimal fraction, for example remainder 3 is often translated as point 3 or one third and vice versa. The **Int ÷** key and the **÷** key (see Figure 3) can form part of a task to help children overcome this misconception.

It is also possible to set the calculator to offer a fraction answer to the same question. Direct children to the **mode** key and then select the **n/d** option in the display. Key in 27 ÷ 6 **Enter** and the display will show 4 and 5 tenths. Simplify if you wish via the **Simp** and **Enter** keys (see Figure 3). Discussion and comment can be focussed on the similarities and differences in the answer displays. For some children it is possible to connect the remainder with the divisor and the fraction answer. For example, 4 r 3 can also be written as 4 and 3/6. This can be connected via equivalent fractions to a half (3/6 = 1/2). The same discussion can be held with the second example in the chart.

Calculation	Answer with Integer divide Int ÷	Answer with Divide ÷	Answer with Divide fraction	Comment
27 ÷ 6	4 r 3	4.5	4 5/10 4 1/2	
46 ÷ 4	11 r 2	11.5	11 5/10 11 1/2	

Figure 8. Remainders, decimals and fractions chart.

Fractions to decimals and back again

The TI-15 calculator is a useful addition to teaching materials for developing fraction knowledge and understanding as it has a number of functions such as the ability to fix the number of decimal places in a number, for example the keys **Fix** and **0.01** will round the result to the nearest hundredth. The calculator also has a function to convert common fractions to decimal fractions and vice versa.

Ask children to press **Fix** and set the calculator to **0.01** by pressing the named key. They then follow this by keying **1.2345** and **Enter** and recording the display answer of **1.24**. After discussing the answer, set children the task of finding other decimal numbers that round to 1.24.

The calculator's ability to convert decimals to fractions quickly allows for many examples to be generated once children are shown how to operate the function. It is possible to help children connect frequently occurring decimals and fractions as they compile a table. Such connections are useful for mental computation as

they allow children to switch to whichever form is more effective for the calculation. Later connections to commonly used percents is also helpful.

Decimal	Fraction	Comment
0.5	5 tenths or 1 half	
0.25	25 hundredths or 1 fourth	
0.75	75 hundredths or 3 fourths	

Figure 9. Decimal to fraction table.

Decimals may be converted to fractions on the TI-15 by following these keying steps:

.5 Enter F–D

Further fraction families can be explored, for example thirds, fifths and eighths. If the fraction column is filled, children will have to reverse the conversion process by entering the fraction first and then using the **F–D** conversion key.

Decimal	Fraction	Comment
	1 third	
0.125		
	6 eighths	

Figure 10. Fraction decimal connections.

Some fractions, for example one third, will present children with recurring decimals. This is a useful area for discussion of recurring but also of the limitations and features of the calculator. For example, 1 ÷ 3 (a third) provides a decimal fraction of 0.3333. If this answer is multiplied by 3 the starting number of 1 should be reached. It does on the TI-15 but does it on other calculators?

More able children in the class can be asked to find more examples of recurring fractions, for example one ninth. The availability of the calculator makes the generation of examples for this task easy for the children.

Conclusions

The calculator used as a learning tool can provide children with challenging insights into understanding fractions and decimals. As part of a teaching package for learning about decimal and fraction ideas, the TI-15 model of calculator can add motivation, understanding and a real-world relevance to an often misunderstood area of mathematics. With appropriate reflection and thinking, it may be possible to remove the demons from denominators for many children.

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Len Sparrow
Curtin University, WA
<l.sparrow@curtin.edu.au>
Paul Swan
Edith Cowan University, WA
<p.swan@ecu.edu.au>

APMC